

# The Nasdaq crash of April 2000: Yet another example of log-periodicity in a speculative bubble ending in a crash

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**Abstract.** The Nasdaq Composite fell another  $\approx 10\%$  on Friday the 14<sup>th</sup> of April 2000 signaling the end of a remarkable speculative high-tech bubble starting in spring 1997. The closing of the Nasdaq Composite at 3321 corresponds to a total loss of over 35% since its all-time high of 5133 on the 10<sup>th</sup> of March 2000. Similarities to the speculative bubble preceding the infamous crash of October 1929 are quite striking: the belief in what was coined a “New Economy” both in 1929 and presently made share-prices of companies with three digits price-earning ratios soar. Furthermore, we show that the largest draw downs of the Nasdaq are outliers with a confidence level better than 99% and that these two speculative bubbles, as well as others, both nicely fit into the quantitative framework proposed by the authors in a series of recent papers.

**PACS.** 01.75.+m Science and society – 02.50.-r Probability theory, stochastic processes, and statistics – 89.90.+n Other topics of general interest to physicists

## 1 Introduction

A series of recent papers [1–6] have presented increasing evidence that market crashes as well as large corrections are often preceded by speculative bubbles with two main characteristics: a power law acceleration of the market price decorated with log-periodic oscillations. Here, “log-periodic” refers to the fact that the oscillations are periodic in the logarithm of the time-to-crash. Specifically, it has been demonstrated that the equation

$$F(t) = A + B(t_c - t)^z + C(t_c - t)^z \cos(\omega \ln(t_c - t) - \phi) \quad (1)$$

remarkably well quantifies the time-evolution of the bubble in terms of the price ending with a crash or large correction at a time close to  $t_c$ . This equation corresponds to a first order Fourier expansion of the general power law solution to a “renormalization equation”

$$\frac{dF(t)}{d \ln(t_c - t)} = z + i\omega \quad (2)$$

around the “critical point”  $t_c$ . Quite remarkable, for all the bubbles in the most liquid markets, *e.g.*, USA, Hong-Kong and the Foreign Exchange Market, the log-frequency  $\omega/2\pi$  have consistently been close to 1. Within the framework of power laws with complex exponents, or equivalently discrete scale invariance [7], this corresponds to a preferred scaling ratio  $\lambda \approx e \approx 2.7$ : the local period of the log-periodic oscillations decreases according to a geometrical series with the ratio  $\lambda$ . For a range of emergent markets, larger fluctuations were seen in the value of  $\lambda$ , but the statistics resulting from over twenty bubbles were quite consistent with that of the larger markets [6]. In contrast, the “universality” of the value of the real part of the exponent quantifying the acceleration in the price has not been established. From a theoretical view point, this is not surprising: a rational expectation model of bubbles and crashes show that depending on whether the size of the crash is proportional to the price itself or that of the increase due to the bubble, either the logarithm of the price or the price itself is the correct quantity characterising the bubble [3]. Another more technical reason for the larger fluctuations in the exponent  $z$  comes from the well-known sensitivity in the determination of critical exponents due to finite-size-effects as well as errors in the determination of the value of the critical point  $t_c$ .

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The question is whether we, in an objective and non-arbitrary manner, can define a crash and hence when we should expect equation (1) to be a good description of the preceding bubble? This is the subject of the next section.

## 2 Crashes are outliers

It is well-known that the distributions of stock market returns exhibit “fat tails”. For example, a 5% daily loss in the Dow Jones Industrial Average occurs approximately once every two years while the Gaussian framework would predict one such loss in about a thousand year. Furthermore, the unconditional volatility on various emergent markets is much higher than on developed equity markets [8]. These empirical observations has led to the development of more sophisticated models than the Gaussian, for instance involving power law tails [9–13] or stretched exponentials [14] as well as models allowing for non-stationary of volatility such as ARCH and GARCH models [15], which better reproduces the statistics of the market fluctuations. Crashes on the other side are the most extreme events and there are two possibilities to describe them:

1. The distribution of returns is stationary and the extreme events can be extrapolated as lying in its far tail. Within this point of view, recent works in finance and insurance have recently investigated the relevance of the body of theory known as Extreme Value Theory to extreme events and crashes [16–18].
2. Crashes cannot be accounted for by an extrapolation of the distribution of smaller events to the regime of extremes and belong intrinsically to another regime, another distribution, and are thus outliers.

In order to see which one of these two descriptions is the most accurate, a statistical analysis of market fluctuations [19] was performed. Instead of looking at the usual “one-point statistics” in terms of the distribution of returns, higher order correlations were included by instead considering so-called *draw downs*. A draw down is defined as a persistent decrease in the index over consecutive days. Specifically, the daily closing of the Dow Jones was considered disregarding occasional single upwards movements of less than 1%. It was established that the distribution of draw downs of the Dow Jones Average daily closing from 1900 to 1993 is well approximated by an exponential distribution with a decay constant of about 2%. (As we will soon see the decay is actually slower than that of an exponential.) This exponential distribution holds only for draw downs smaller than about 15%. In other words, this means that all draw downs of amplitudes of up to approximately 15% are well approximated by the same exponential distribution with characteristic scale 2%. This characteristic decay constant means that the probability of observing a draw down larger than 2% is about 37%. Following hypothesis 1 and extrapolating this description to, *e.g.*, the three largest crashes on the USA market in this century (1914, 1929 and 1987) yields a recurrence time

of about 50 centuries for *each single* crash. In reality, the three crashes occurred in less than one century.

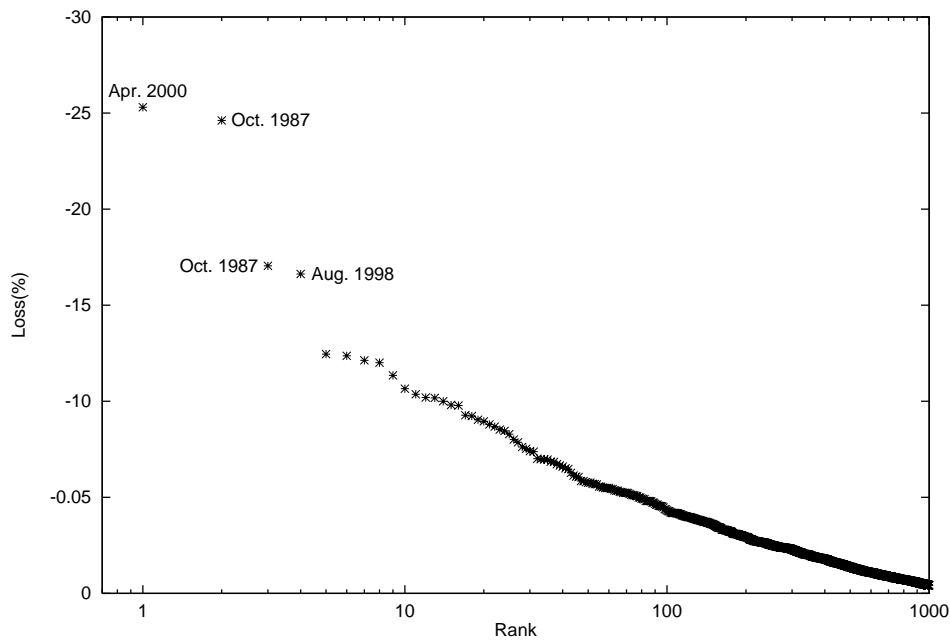
As an additional null-hypothesis, 10 000 synthetic data sets, each covering a time-span close to a century hence adding up to about  $10^6$  years, have been generated using a GARCH(1,1) model estimated from the true index with a t-student distribution with four degrees of freedom [15]. This model includes both non-stationarity of volatilities and fat tail nature of the price returns. In conclusion, our analysis [5] showed that in approximately one million years of heavy tail “Garch-trading”, with a reset every century, *never* did three crashes similar to the three largest observed in the true Dow Jones Index occur in a single “Garch-century”. Of course, these simulations do not prove that our model is the correct one, only that one of the standard models of the “industry” (which makes a reasonable null hypothesis) is utterly unable to account for the stylized facts associated with large financial crashes. What it suggests is that different mechanisms are responsible for large crashes and that hypothesis 2 is the correct description of crashes.

A similar picture has been found for the Nasdaq Composite. In Figure 1, we see the rank ordering plot of “pure” draw downs, *i.e.*, no threshold (see [19] for a brief discussion of the effect of thresholds), since the establishment of the index in 1971 until 18 April 2000. Recall that the rank ordering plot, which is the same as the (complementary) cumulative distribution with axis interchanged, puts emphasis on the largest events. Again, we see that the four largest events are not situated on a continuation of the distribution of smaller events: the jump between rank 4 and 5 in magnitude is  $> 33\%$  whereas the corresponding jump between rank 5 and 6 is  $< 1\%$  and this remains true for higher ranks. This means that, for draw downs less than 12.5%, we have a more or less “smooth” curve and then a  $> 33\%$  gap(!) to rank 3 and 4. The four events are according to rank the crash of April 2000 analysed here, the crash of October 1987, a  $> 17\%$  “after-shock” related to the crash of October 1987 and a  $> 16\%$  drop related to the “slow crash” of August 1998.

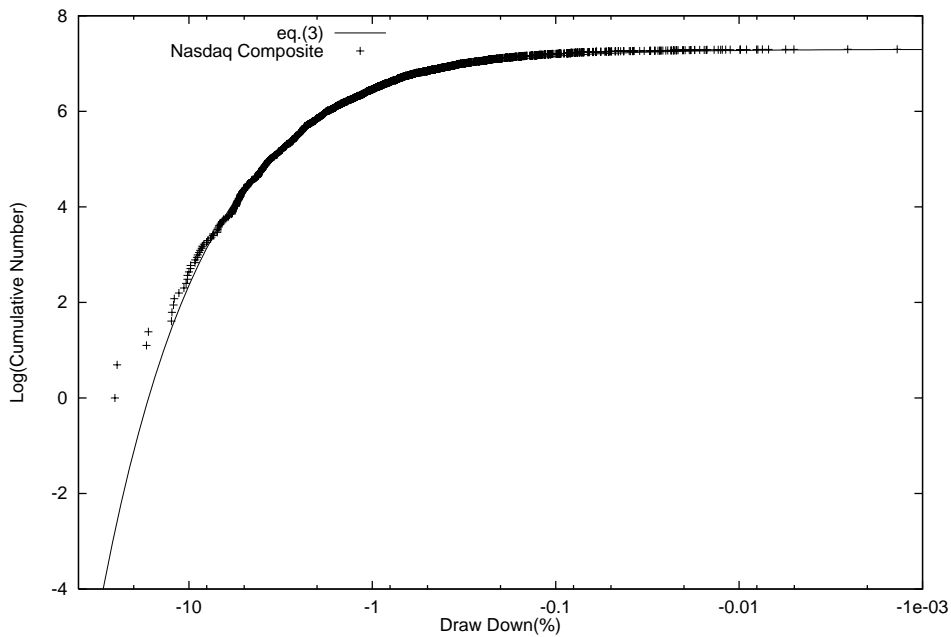
In order to quantify the cumulative distribution of draw downs  $N(x)$ , we compare it with a stretched exponential

$$N(x) \approx a \exp(-bx^c) \quad (3)$$

as null-hypothesis [14], see Figure 2 and caption. Confirming the result from the rank ordering, we see that the stretched exponential captures well the distribution except for the four largest events. Furthermore, it is clear from Figure 2 that the distribution is not that of a power law which would be qualified as a straight line in this log-log plot. One could perhaps argue that the tail of the cumulative distribution tends to become linear in this log-log representation; however, this observation is based on an interval smaller than half-a-decade. In addition, in the case of the Dow Jones data shown in Figure 4, the tail is even fatter than a power law with an upward convexity in the log-log representation. The principle of parsimony leads us to prefer not to assume any distribution



**Fig. 1.** Rank ordering of draw downs in the Nasdaq Composite since its establishment in 1971 until 18 April 2000.

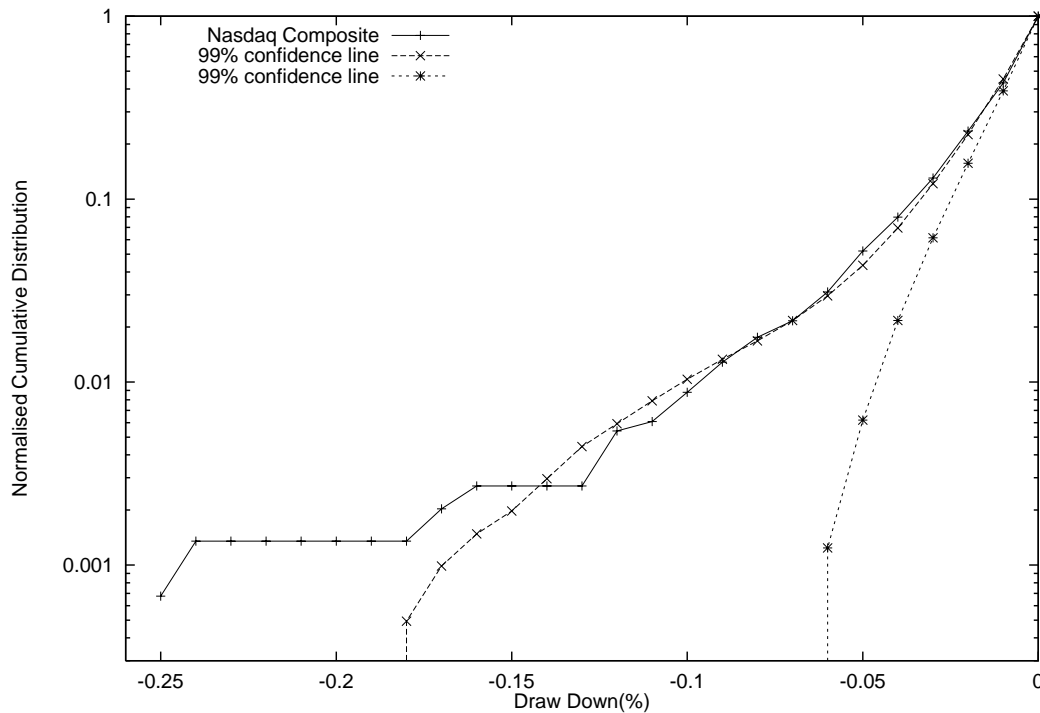


**Fig. 2.** Natural logarithm of the cumulative distribution of draw downs  $N(x)$  in the Nasdaq Composite since its establishment in 1971 until 18 April 2000. The fit is  $\ln(N) = \ln(1479) - 29.0x^{0.77}$  assuming that the distribution follows a stretched exponential  $N(x) = a \exp(-bx^c)$ . Here  $a = 1479$  is the total number of draw downs. The exponent  $c \approx 0.8$  is compatible with values previously found in other markets [14,27]. Note that, in this representation, a powerlaw distribution would qualify as a straight line.

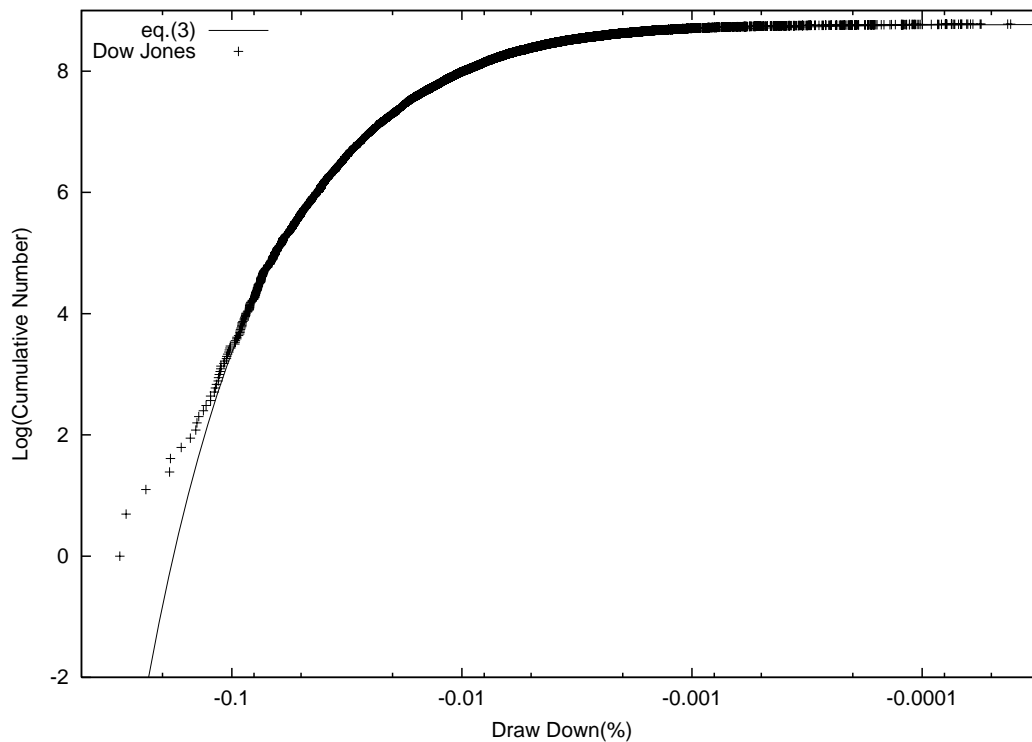
in the tail and only conclude about the fact that the few largest events are clearly taken from a different distribution. Indeed, if we extrapolate the curve to larger events, we get that, in the  $\approx 30$  years of the existence of the Nasdaq Composite, we should have observed 0.09 draw downs above 24%, whereas in reality we have observed 2.

To further establish the statistical confidence with which we can conclude that the four largest events are outliers, we have reshuffled the daily returns 1000 times

and hence generated 1000 synthetic data sets. This procedure means that the synthetic data will have exactly the same distribution of daily returns. However, higher order correlations apparently present in the largest draw downs are destroyed by the reshuffling. This surrogate data analysis of the distribution of draw downs has the advantage of being *non-parametric*, *i.e.*, independent of the quality of fits with a model such as the stretched exponential or the power law. We will now compare the distribution of draw



**Fig. 3.** Normalised cumulative distribution of draw downs in the Nasdaq Composite since its establishment in 1971 until 18 April 2000. The 99% confidence lines are estimated from the synthetic tests described in Section 2.



**Fig. 4.** Natural logarithm of the cumulative distribution of draw downs  $N(x)$  in the Dow Jones since 1900 until 2 May 2000. The fit is  $\ln(N) = \ln(6469) - 36.3x^{0.83}$  assuming that the distribution follows a stretched exponential  $N(x) = a \exp(-bx^c)$ . Here  $a = 6469$  is the total number of draw downs. The exponent  $c \approx 0.8$  is in remarkable agreement with the value found in the Nasdaq Composite, see caption of Figure 2. The outliers to the fit are according to rank the crash of October 1987, the crash in 1914 related to the outbreak of the First World War, the crash of October 1929, two  $> 18\%$  crashes in 1932 and 1933 respectively and two  $> 15\%$  “aftershocks” related to the October 1929 crash. Note that, in this representation, a powerlaw distribution would qualify as a straight line.

downs both for the real data and the synthetic data. With respect to the synthetic data, this can be done in two complementary ways. In Figure 3, we see the distribution of draw downs in the Nasdaq Composite compared with the two lines constructed at the 99% confidence level for the entire *ensemble* of synthetic draw downs, *i.e.* by considering the individual draw downs as independent: for any given draw down, the upper (resp. lower) confidence line is such that 5 of the synthetic distributions are above (below) it; as a consequence, 990 synthetic times series out of the 1000 are within the two confidence lines for any draw down value which define the typical interval within which we expect to find the empirical distribution.

Two features are apparent in Figure 3. First, the distribution of the true data breaks away from the 99% confidence intervals at  $\approx 15\%$ , showing that the four largest events are indeed outliers in this sense. In addition, the empirical distribution of draw downs is systematically found close to the upper confidence boundary, with an upward curvature described by the apparent stretched exponential equation (3), for values less than 15%. In contrast, the median value between the two confidence lines is approximately linear in this semi-logarithmic representation, qualifying an exponential distribution as expected for uncorrelated daily returns (see Appendix 1 of [5]). The upward curvature of the distribution of draw downs and its closeness to the upper confidence line thus signals a subtle dependence between consecutive returns.

A more sophisticated analysis is to consider each synthetic data set *separately* and calculate the *conditional probability* of observing a given draw down given some prior observation of draw downs. This gives a more precise estimation of the statistical significance of the outliers, because the previously defined confidence lines neglect the correlations created by the ordering process which is explicit in the construction of a cumulative distribution.

Out of the 10000 synthetic data sets, 776 had a single draw down larger than 16.5%, 13 had two draw downs larger than 16.5%, 1 had three draw downs larger than 16.5% and none had 4 (or more) draw downs larger than 16.5% as in the real data. This means that given the distribution of returns, by chance we have a  $\approx 8\%$  probability of observing a draw downs larger than 16.5%, a  $\approx 0.1\%$  probability of observing two draw downs larger than 16.5% and for all practical purposes zero probability of observing three or more draw downs larger than 16.5%. Hence, the probability that the largest four draw downs observed for the Nasdaq could result from chance is outside a 99.99% confidence interval. As a consequence we are lead to conclude that the largest market events are to be characterised by the presence of higher order correlations in contrast to what is observed during “normal” times.

Performing a fit with a stretched exponential on the cumulative distribution of “pure” draw downs in the Dow Jones index since 1900 until May 2000, *i.e.*, no threshold as for the Nasdaq Composite, gives a remarkably similar result as that of the Nasdaq Composite, see Figure 4 and caption, both with respect to the exponent as well as the

point where the data “breaks away” from the fit. If we again extrapolate the fit to the largest events, we get that 0.12 draw downs above 23.5% should have occurred in the last century whereas we in fact have observed 3.

This analysis confirms the conclusion from the previous analysis of the Dow Jones that draw downs larger than  $\approx 15\%$  are to be considered as outliers with high probability. It is interesting that the same amplitude of  $\approx 15\%$  is found for both markets considering the much larger daily volatility of the Nasdaq Composite. This may result from the fact that, as we have shown, very large draw downs are more controlled by transient correlations than by the amplitude of daily returns.

The presented statistical analysis of the Dow Jones Average and the Nasdaq Composite suggests that large crashes *are* special and that precursory patterns may exist decorating the speculative bubble preceding the crash. As we have discussed in the preceding section, such precursory patterns do exist prior to these outliers and can be quantified by equation (1). It is the subject of the next section to quantify these precursory patterns for the Nasdaq bubble starting in spring 1997 and ending April 2000 and to compare with the results previously obtained for large crashes.

### 3 The current crash

With the low of 3227 on the 17 April, the Nasdaq Composite lost over 37% of its all-time high of 5133 reached on the 10<sup>th</sup> of March this year. The Nasdaq Composite consists mainly of stock related to the so-called “New Economy”, *i.e.*, the Internet, software, computer hardware, telecommunication ... A main characteristic of these companies is that their price-earning-ratios (P/E’s), and even more so their price-dividend-ratios, often come in three digits<sup>1</sup>. Opposed to this, so-called “Old Economy” companies, such as Ford, General Motors and Daimler-Chrysler, have P/E  $\approx 10$ . The difference between “Old Economy” and “New Economy” stocks is thus the expectation of *future earnings* as discussed in [20]: investors expect an enormous increase in for example the sale of Internet and computer related products rather than in car sales and are hence more willing to invest in Cisco rather than in Ford notwithstanding the fact that the earning-per-share of the former is much smaller than for the later. For a similar price per share (approximately \$60 for Cisco and \$55 for Ford), the earning per share is \$0.37 for Cisco compared to \$6.0 for Ford (Cisco has a total market capitalisation of \$395 billions (close of April, 14, 2000) compared to \$63 billions for Ford). In the standard fundamental valuation formula, in which the expected return of a company is the sum of the dividend return and of the growth rate, “New Economy” companies are supposed

<sup>1</sup> VA LINUX to be discussed below actually has a *negative* Earning/Share of  $-1.68$ . Yet they are currently traded around \$40 per share which is close to the price of Ford in early March 2000.

to compensate for their lack of present earnings by a fantastic potential growth. In essence, this means that the bull market observed in the Nasdaq the last three years until recently is fueled by expectations of increasing future earnings rather than economic fundamentals: the price-to-dividend ratio for a company such as Lucent Technologies (LU) with a capitalization of over \$300 billions prior to its crash on the 5 January 2000 is over 900 which means that you get a higher return on your checking account(!) unless the price of the stock increases. Opposed to this, an “Old Economy” company such as DaimlerChrysler gives a return which is more than thirty times higher. Nevertheless, the shares of Lucent Technologies rose by more than 40% during 1999 whereas the share of DaimlerChrysler declined by more than 40% in the same period. Truly surrealistic is the fact that the recent crashes of IBM, LU and Procter & Gamble (P&G) correspond to a loss equivalent to many countries state budget! And this is usually attributed to a “business-as-usual” corporate statement of a slightly revised smaller-than-expected earnings!

These considerations makes it clear that it is the *expectation* of future earnings that motivates the average investor rather than present economic reality, thus creating a speculative bubble. History provides many examples of bubbles driven by unrealistic expectations of future earnings followed by crashes [21]. The same basic ingredients are found repeatedly: fueled by initially well-founded economic fundamentals, investors develop a self-fulfilling enthusiasm by an imitative process or crowd behavior that leads to the building of “castles in the air”, to paraphrase Malkiel [22]. Furthermore, the causes of the crashes on the US markets in 1929, 1987, 1998 and present belongs to the same category, the difference being mainly in which sector the bubble was created: in 1929, it was utilities; in 1987, the bubble was supported by a general deregulation of the market with many new private investors entering the market with very high expectations with respect to the profit they would make; in 1998, it was an enormous expectation to the investment opportunities in Russia that collapsed; as for the present, it is the extremely high expectations to the Internet, telecommunication etc. that has fueled the bubble. The IPO’s (initial public offerings) of many Internet and software companies has been followed by a mad frenzy where the price of the share has soared during the first few hours of trading. An excellent example is VA LINUX SYSTEMS whose \$30 IPO price increased a record 697 percent to close at \$239.25 on its opening day 9 Dec. 1999, only to decline to \$28.94 on the 14 April 2000.

In Figure 5, we see the logarithm of the Nasdaq Composite fitted with equation (1). The data interval to fit was identified using the same procedure as for the other crashes: the first point is the lowest value of the index prior to the onset of the bubble and the last point is that of the all-time high of the index. There exists some subtlety with respect to identifying the onset of the bubble, the end of the bubble being objectively defined as the date where the market reached is maximum. A bubble signifies an acceleration of the price. In the case of Nasdaq, it tripled from 1990 to 1997. However, the increase was a about factor 4

in the 3 years preceding the current crash thus defining an “inflection point” in the index. In general, the identification of such an “inflection point” is quite straightforward on the most liquid markets whereas this is not the case for the emergent markets. With respect to details of the methodology of the fitting procedure, we refer the reader to [4,23].

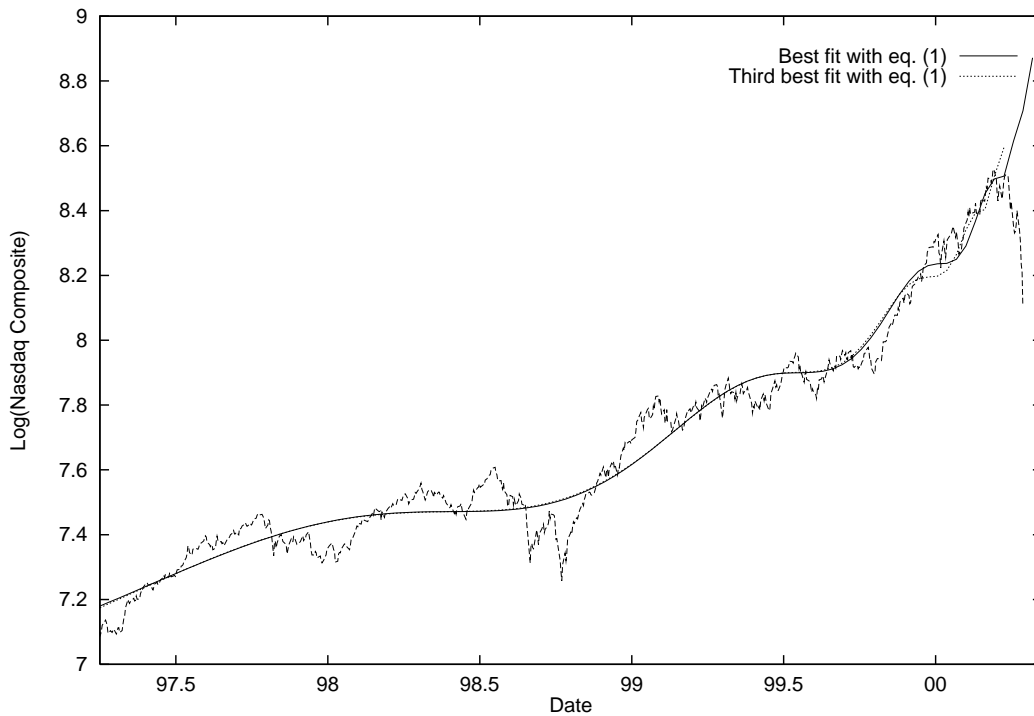
Three fits were obtained with similar parameter values for the best and third best fit, whereas the second best fit had a rather small value for  $z \approx 0.08$  and a rather high value for  $\omega \approx 7.9$  compared with previous results and is not shown. The values obtained for the best and third best fit are  $\omega \approx 7.0$  and  $\omega \approx 6.5$ ,  $z \approx 0.27$  and  $z \approx 0.39$ ,  $t_c \approx 2000.34$  and  $t_c \approx 2000.25$ , respectively. These results pointed to a crash occuring between the 31 of March 2000 and 2 May 2000 and have now been confirmed by the recent market event.

## 4 Prediction

An obvious question concerns the predictive power of equation (1). In the present case, the last point used in the fitted data interval was that of March 10, 2000. The predicted time of the crash was as mentioned 2 May for the best fit and 31 March for the third best fit. Except for slight gains on 31 March and 5, 6 and 7 April, the closing of the Nasdaq Composite has been in continuous decline since the 24 March and lost over 25 % in the week ending on Friday the 14 April. Consequently, the crash occured approximately in between the predicted date of the two fits. The corresponding dates for the 1929, 1987 and 1998 crashes on Wall Street and the 1987, 1994 and 1997 crashes on the Hong-Kong stock exchange as well as the collapse of the US\$ in 1985 are shown in Table 1 for comparison.

**Table 1.**  $t_c$  is the critical time predicted from the fit of the financial time series to the equation (1). The other parameters  $z$ ,  $\omega$  and  $\lambda$  of the fit are also shown. The fit is performed up to the time  $t_{\max}$  at which the market index achieved its highest maximum before the crash.  $t_{\min}$  is the time of the lowest point of the market after the maximum. The percentage drop is calculated from the total loss from  $t_{\max}$  to  $t_{\min}$ .

crash	$t_c$	$t_{\max}$	$t_{\min}$	% drop	$z$	$\omega$	$\lambda$
1929 (DJ)	30.22	29.65	29.87	47%	0.45	7.9	2.2
1985 (DM)	85.20	85.15	85.30	14%	0.28	6.0	2.8
1985 (CHF)	85.19	85.18	85.30	15%	0.36	5.2	3.4
1987 (S&P)	87.74	87.65	87.80	30%	0.33	7.4	2.3
1987 (H-K)	87.84	87.75	87.85	50%	0.29	5.6	3.1
1994 (H-K)	94.02	94.01	94.04	17%	0.12	6.3	2.7
1997 (H-K)	97.74	97.60	97.82	42%	0.34	7.5	2.3
1998 (S&P)	98.72	98.55	98.67	19.4%	0.60	6.4	2.7
1999 (IBM)	99.56	99.53	99.81	34%	0.24	5.2	3.4
2000 (P&G)	00.04	00.04	00.19	54%	0.35	6.6	2.6
2000 (Nasdaq)	00.34	00.22	00.29	37%	0.27	7.0	2.4



**Fig. 5.** Best (r.m.s.  $\approx 0.061$ ) and third best (r.m.s.  $\approx 0.063$ ) fits with equation (1) to the natural logarithm of the Nasdaq Composite. The parameter values of the fits are  $A \approx 9.5$ ,  $B \approx -1.7$ ,  $C \approx 0.06$ ,  $z \approx 0.27$ ,  $t_c \approx 2000.33$ ,  $\omega \approx 7.0$ ,  $\phi \approx -0.1$  and  $A \approx 8.8$ ,  $B \approx -1.1$ ,  $C \approx 0.06$ ,  $z \approx 0.39$ ,  $t_c \approx 2000.25$ ,  $\omega \approx 6.5$ ,  $\phi \approx -0.8$ , respectively.

We see that, in all 9 cases, the market crash started at a time between the date of the last point and the predicted  $t_c$ . And with the exception of the October 1929 crash and using the third best fit of the present crash (this fit had  $\omega/2\pi \approx 1$ ), in all cases the market ended its decline less than approximately one month after the predicted  $t_c$ . These results indeed suggest that predictions of crashes with equation (1) is indeed possible.

Furthermore, the crashes of the shares of IBM, LU and P&G, *i.e.*, three of the largest US companies, may be taken as precursors of a pending crash signifying how unstable the market actually was in the months preceding the current crash. Quite remarkably, two of these three company crashes were also preceded by a speculative bubble with the same characteristics as previously seen on the market as a whole, see Figures 7, 6 and Table 1.

Of course, the results presented here does not mean that we have publicly predicted the April 2000 crash of the Nasdaq Composite. This has neither been the purpose. What the analysis presented above shows is that equation (1) has predictive power. Furthermore, from a purely scientific point of view, it is the *observation* and the *comparison* between the observation and the predictions of the model which carry meaning.

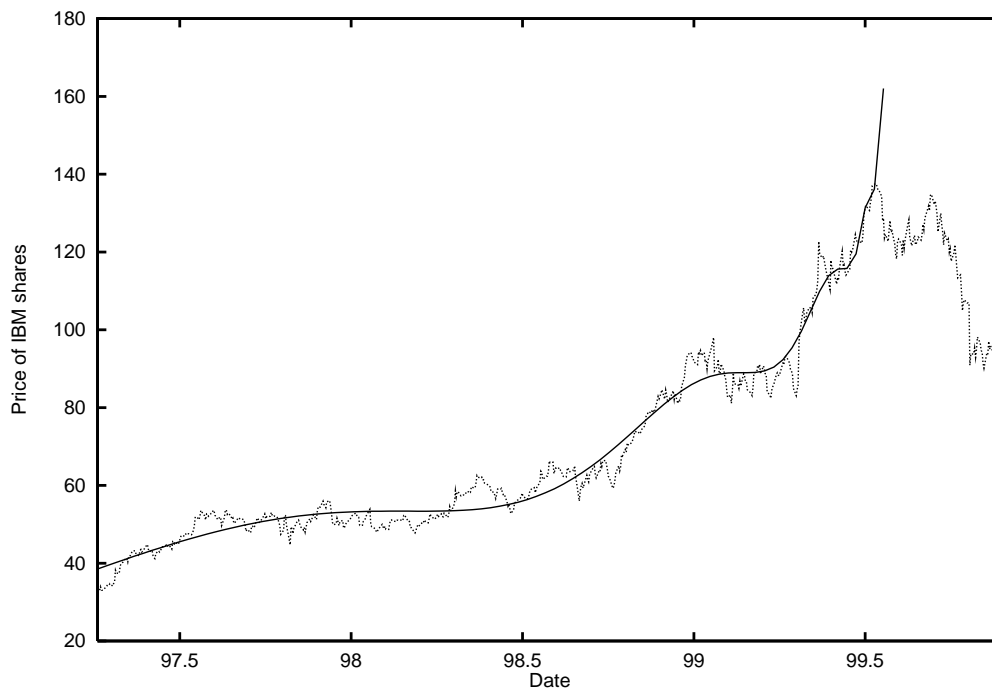
## 5 False alarms

Not all speculative bubbles end in a crash. Hence, the question about false alarms enters naturally. We have

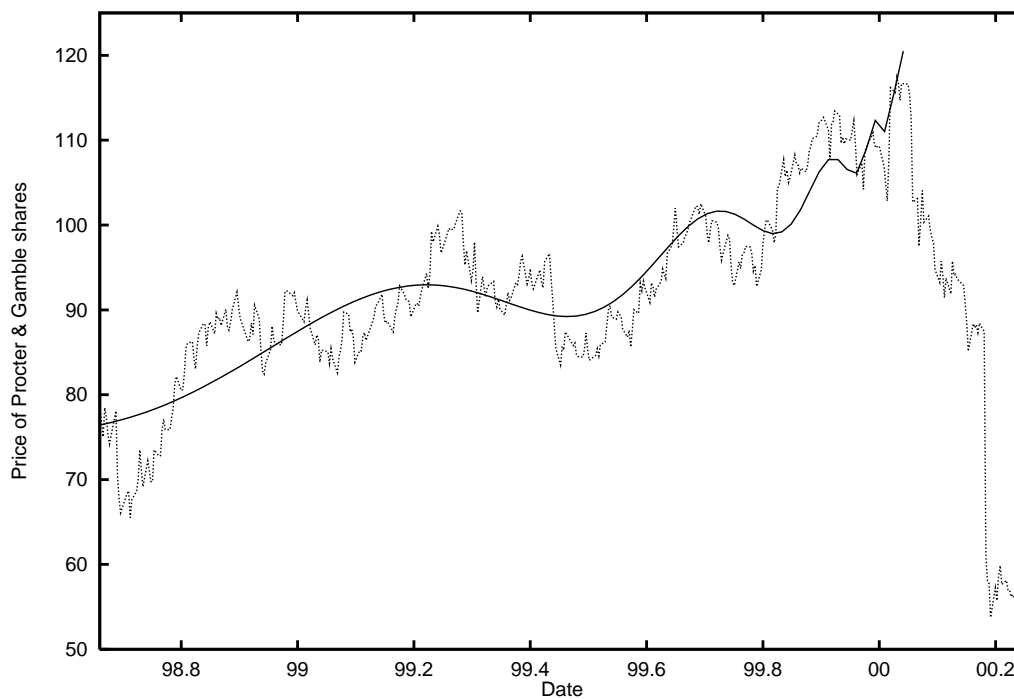
twice identified a log-periodic power law bubble signaling a crash where the market in fact *did not* crash according to the definition presented in Section 2. The first attempt was in October 1997 where the market dropped only 7% [24] and quickly recovered (see also [25]). The second attempt was in October last year when the world markets were sent into turmoil by a speech by Alan Greenspan and the Dow Jones for the first time since 8 April 1999 dipped below 10000 on the 15 and 18 October 1999. However, the market did not crash and instead quickly recovered. These two examples of bubbles landing more or less smoothly are completely consistent with the theory of rational bubbles and crashes developed in [4]. This also illustrates the difficulties involved in a crash-prediction scheme using equation (1): according to the theory, the critical time  $t_c$  is not necessarily the time of the crash, only its most probable time; in addition, there is a finite probability that the bubble ends without crashing. We are currently investigating how to extend the methodology in order to increase the reliability of the model in terms of predictions.

## 6 Conclusion

Here, we have provided yet an example of a speculative bubble with power law acceleration and log-periodic oscillations ending in a crash/major correction, *i.e.*, that of the Nasdaq Composite starting in spring 1997 ending in late March/early April 2000. The log-frequency of these



**Fig. 6.** Best (r.m.s.  $\approx 3.7$ ) fit with equation (1) to the price of IBM shares. The parameter values of the fits are  $A \approx 196$ ,  $B \approx -132$ ,  $C \approx -6.1$ ,  $z \approx 0.24$ ,  $t_c \approx 99.56$ ,  $\omega \approx 5.2$  and  $\phi \approx 0.1$



**Fig. 7.** Best (r.m.s.  $\approx 4.3$ ) fit with equation (1) to the price of Procter & Gamble shares. The parameter values of the fit are  $A \approx 124$ ,  $B \approx -38$ ,  $C \approx 4.8$ ,  $z \approx 0.35$ ,  $t_c \approx 2000.04$ ,  $\omega \approx 6.6$  and  $\phi \approx -0.9$ .



oscillations is in remarkable agreement with what has been obtained previously on a wide range of markets [4,6].

The present analysis of these market phases emphasizes a collective behavior of investors, leading to a fundamental ripening of the markets towards an instability. This must be contrasted with the endeavor of economists and analysts who search for contemporary news to explain the events. For instance, the August 1998 crash was often attributed to a devaluation of the ruble and to events on the Russian political scene. While we do not underestimate the effect of “news”, we observe that markets are constantly “bombarded” by news and it will always be possible to attribute the crash to a specific one, *after the fact*. In contrast, we view their reactions more often than not as reflecting their underlying stability (or instability). In the case of the August 1998 crash, the market was ripe for a correction and the “news” made it occur. If nothing had occurred on the Russian scene, we have proposed [4] that other news would have triggered the event anyway, within a time scale of about a month, which seems to be the relevant lifetime of a market instability associated with the burst of a bubble. With respect to the present Nasdaq crash, undoubtedly, analysts will forge post-mortem stories linking it in part with the effect of the crash of Microsoft Inc. resulting from the breaking of negotiations during the week-end of April 1st with the US federal government on the antitrust issue. Again, we see the Nasdaq crash as the natural death of a speculative bubble, anti-trust or not, the results presented here strongly suggest that the bubble would have collapsed anyway.

However, according to our analysis (see the probabilistic model of bubbles in [3–5]), the exact timing of its death is not fully deterministic and allows for stochastic influences, but within the remarkably tight bound of about one month.

We have also discussed the possibility of using the proposed framework, specifically equation (1), in order to predict when the market will exhibit a crash/major correction. Our analysis not only points to a predictive potential but also that false alarms are difficult to avoid due to the underlying nature of speculative bubbles.

A fundamental remaining question concerns the use of a reliable crash prediction scheme. Assume that a crash prediction is issued stating that a crash will occur  $x$  weeks from now. At least three different scenarios are possible:

- Nobody believes the prediction which was then futile and, assuming that the prediction was correct, the market crashes<sup>2</sup>.
- Everybody believes the warning, which causes panic and the market crashes as consequence. The prediction hence seems self-fulfilling.

<sup>2</sup> One may consider this as a victory for the “predictors” but as we have experienced in relation to our quantitative prediction of the change in regime of the Nikkei index [26], this would only be considered by some critics just another “lucky one” without any statistical significance (see [28] for an alternative Bayesian approach).

- Enough people believe that the prediction *may* be correct and the steam goes off the bubble. The prediction hence disproves itself.

None of these scenarios are attractive. In the first two, the crash is not avoided and in the last scenario the prediction disproves itself and as a consequence the theory looks unreliable. This seems to be the unescapable lot of scientific investigations of systems with learning and reflective abilities, in contrast with the usual inanimate and unchanging physical laws of nature. Furthermore, this touches the key-problem of scientific responsibility. Naturally, scientists have a responsibility to publish their findings. However, when it comes to the practical implementation of those findings in society, the question becomes considerably more complex.

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